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Hypernuclei in Halo/Cluster Effective Field Theory

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The light double Λ hypernuclei, ${}_{\Lambda\Lambda}^4\text{H}$ and ${}_{\Lambda\Lambda}^6\text{He}$, are studied as three-body $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ cluster systems in halo/cluster effective field theory at leading order. We find that the $\Lambda\Lambda d$ system in spin-0 channel does not exhibit a limit cycle whereas the $\Lambda\Lambda d$ system in spin-1 channel and the $\Lambda\Lambda\alpha$ system in spin-0 channel do. The limit cycle is associated with the formation of bound states, known as Efimov states, in the unitary limit. For the $\Lambda\Lambda d$ system in the spin-0 channel we estimate the scattering length a_0 for S -wave Λ hyperon-hypertriton scattering as $a_0 = 16.0 \pm 3.0$ fm. We also discuss that studying the cutoff dependences in the $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ systems, the bound state of ${}_{\Lambda\Lambda}^4\text{H}$ is not an Efimov state but formed due to a high energy mechanism whereas that of ${}_{\Lambda\Lambda}^6\text{He}$ may be regarded as an Efimov state.

Keywords: Few-body systems; Hypernuclei; Renormalization group evolution of parameters.

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1. Introduction

Light double Λ hypernuclei are exotic few-body systems that provide an opportunity to investigate the $SU(3)$ flavor baryon-baryon interaction in the strangeness -2 channel.^{1–3} They are also expected to play a key role in resolving the long-standing puzzle of the existence of the H dibaryon,⁴ which has recently been studied by lattice QCD simulations.^{5,6} Meanwhile $SU(3)$ flavor baryon-baryon potentials have been obtained in chiral effective theory, hyperon-nucleon potentials up to next-to-leading order⁷ and hyperon-hyperon potentials at leading order (LO)² assuming $SU(3)$ flavor symmetry except for the masses of mesons and baryons. The main difficulty of the study is that one does not have an adequate number of experimental data to accurately determine all parameters in the potentials. In this situation, it may be worth studying the exotic few-body systems by constructing a simple theory at very low energies which has a handful of parameters.

The first observation of ${}_{\Lambda\Lambda}^6\text{He}$ was reported in the 1960s,⁸ however there have been only a few reports on this light hypernucleus.^{9,10} Among them, a track of ${}_{\Lambda\Lambda}^6\text{He}$ was clearly caught in an emulsion experiment of the KEK-E373 Collaboration,¹⁰

now known as the “NAGARA” event, and the two- Λ separation energy $B_{\Lambda\Lambda}$ of ${}^6_{\Lambda\Lambda}\text{He}$ is estimated as $B_{\Lambda\Lambda} = 6.93 \pm 0.16$ MeV after being averaged with that from the “MIKAGE” event.^{11,12} In addition, the formation of another light double- Λ hypernucleus, ${}^4_{\Lambda\Lambda}\text{H}$, is conjectured in the BNL-AGS E906 experiment.¹³

Theoretical studies for the double Λ hypernuclei mainly aim at extracting information on baryon-baryon interactions in the strangeness sector and searching for new exotic systems for which the value of $B_{\Lambda\Lambda}$ of ${}^6_{\Lambda\Lambda}\text{He}$ plays an important role.^{14,15} In fact, studies of ${}^6_{\Lambda\Lambda}\text{He}$ have been reported with various issues in theory,^{16–20} primarily employing the three-body ($\Lambda\Lambda\alpha$) cluster model. One of those issues is the importance of a mixing of the ΞN channel in the $\Lambda\Lambda$ interaction because the relatively small mass difference between the ΞN and $\Lambda\Lambda$ channels is about 23 MeV (see, e.g., Ref. 19). In theoretical studies for ${}^4_{\Lambda\Lambda}\text{H}$, although the first Faddeev-Yakubovsky calculation showed a negative result,²¹ subsequent theoretical studies^{22–25} predicted the possibility of the ${}^4_{\Lambda\Lambda}\text{H}$ bound state based on the phenomenological $\Lambda\Lambda$ potentials which can describe the bound state of ${}^6_{\Lambda\Lambda}\text{He}$. In this article, we review the results of our recent works on the light double Λ hypernuclei, ${}^4_{\Lambda\Lambda}\text{H}$ and ${}^6_{\Lambda\Lambda}\text{He}$, as three-body $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ cluster systems, studied in halo/cluster effective field theories (EFTs) at LO.^{26,27}

EFTs at very low energies are expected to provide a model-independent and systematic perturbative method where one introduces a high momentum separation scale Λ_H between relevant degrees of freedom in low energy and irrelevant degrees of freedom in high energy for the system in question. Then one constructs an effective Lagrangian expanded in terms of the number of derivatives order by order. Coupling constants appearing in the effective Lagrangian should be determined from available experimental or empirical data. In the study of the three-body systems, we will make use of the cyclic singularities that arise in the solutions for the integral equations in the asymptotic limit.²⁸ Such singularities are renormalized by introducing a suitably large momentum cutoff Λ_c ($\Lambda_c \gtrsim \Lambda_H$) in the loop integrations at the cost of introducing three-body counter terms at LO. Consequently, in order to absorb this cutoff dependence, the corresponding three-body coupling may exhibit a cyclic renormalization group (RG) evolution termed as the limit cycle.²⁹ The cyclic singularities are associated with the occurrence of bound states, known as the Efimov states, in the resonant/unitary limit.³⁰ For a review, see, e.g., Refs. 31, 32 and references therein.

In our study for the three-body systems, we first derive simple equations from the homogeneous part of the integral equations in the asymptotic limit to make an examination whether the systems exhibit a limit cycle or not. When a limit cycle does not appear in the system, it is not necessary to introduce a three-body contact interaction for renormalization at LO. The system would be insensitive to the three-body contact term and can be described by two-body interactions. In the spin-0 channel of the $\Lambda\Lambda d$ system, we shall see that a limit cycle does not appear and thus we can make a prediction on the scattering length a_0 for S -wave Λ hyperon and hypertriton scattering using a parameter of a two-body interaction. When a

limit cycle appears in the system, on the other hand, one needs to introduce a three-body contact interaction at LO. Because the cyclic singularity is related to the occurrence of Efimov states, the bound state emerging in the system can be regarded as an Efimov state. In the study we vary the magnitude of the cutoff within a reasonable range, we investigate its effects on the formation of bound states and discuss whether the bound states of ${}_{\Lambda\Lambda}^4\text{H}$ and ${}_{\Lambda\Lambda}^6\text{He}$ can be Efimov states or not.^a

This work is organized as follows. In Sec. 2, we discuss the scales, the counting rules, and the effective Lagrangian for the $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ systems, and display the expression of the dressed propagators for the two-body composite states and the integral equations for the three-body part. In Sec. 3, we examine the limit cycle behavior of the integral equations in the asymptotic limit for the systems. In Sec. 4 the numerical results and in Sec. 5 a discussion and the conclusions of the work are presented.

2. Formalism

2.1. Scales, counting rules, and effective Lagrangian

To construct an EFT for ${}_{\Lambda\Lambda}^4\text{H}$ as the $\Lambda\Lambda d$ system, we treat the deuteron field as a cluster field, i.e., like an elementary field. Thus the deuteron binding energy, $B_2 \simeq 2.22$ MeV, is chosen as the high energy scale. The large (high momentum) scale Λ_H of the system is the deuteron binding momentum, $\gamma = \sqrt{m_N B_2} \simeq 45.7$ MeV, where m_N is the nucleon mass. We take as the typical momentum (Q) of the reaction the Λ particle separation momentum from the hypertriton, which is defined by $\gamma_{\Lambda d} = \sqrt{2\mu_{\Lambda d} B_\Lambda} \simeq 13.5 \pm 2.6$ MeV, where $\mu_{\Lambda d}$ is the reduced mass of the Λd system and B_Λ is the Λ particle separation energy from the hypertriton, $B_\Lambda^{\text{expt.}} \simeq 0.13 \pm 0.05$ MeV.³³ Then our expansion parameter is $Q/\Lambda_H \sim \gamma_{\Lambda d}/\gamma \simeq 1/3$, which supports our expansion scheme.

To construct an EFT for ${}_{\Lambda\Lambda}^6\text{He}$ as the $\Lambda\Lambda\alpha$ system, we treat the α particle field as an elementary field. The binding energy of the α particle is $B_4 \simeq 28.3$ MeV and its first excited state has the quantum numbers ($J^\pi = 0^+, I = 0$) and the excitation energy of $E_1 \simeq 20.0$ MeV, which is between the energy gap of ${}^3\text{H}-p$ (19.8 MeV) from the ground state energy and that of ${}^3\text{He}-n$ (20.6 MeV). Thus the large momentum scale of the α -cluster theory is $\Lambda_H \simeq \sqrt{2\mu E_1} \sim 170$ MeV where μ is the reduced mass of the $({}^3\text{H}, p)$ system or the $({}^3\text{He}, n)$ system so that $\mu \simeq \frac{3}{4}m_N$. One can see that the mixing of the ΞN channel in the $\Lambda\Lambda$ interaction becomes an irrelevant degree of freedom because of the mass difference ~ 23 MeV. On the other hand, we take the binding momentum of ${}_{\Lambda\Lambda}^5\text{He}$ as the typical momentum scale Q of the theory. The Λ separation energy of ${}_{\Lambda\Lambda}^5\text{He}$ is $B_\Lambda \simeq 3.12$ MeV and thus the binding

^a We refer to an ‘‘Efimov state’’ as a bound state forming due to a singular interaction which exhibits a limit cycle. While each of the systems described in the theory has a scale beyond of which the applicability of the theory breaks down. We examine likelihood of the formation of an Efimov state in the systems by comparing a cutoff value at which an Efimov state emerges with the scale of the theory.

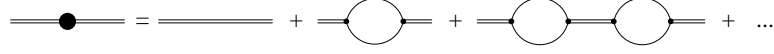
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Fig. 1. Diagrams for dressed dibaryon propagator. In the right hand side, the double solid line represents the bare dibaryon propagator and the single solid line denotes the Λ propagator.

momentum of ${}^5_\Lambda\text{He}$ as the $\Lambda\alpha$ cluster system is $\gamma_{\Lambda\alpha} = \sqrt{2\mu_{\Lambda\alpha}B_\Lambda}$, where $\mu_{\Lambda\alpha}$ is the reduced mass of the $\Lambda\alpha$ system. This leads to $\gamma_{\Lambda\alpha} \simeq 73.2$ MeV and thus our expansion parameter is $Q/\Lambda_H \sim \gamma_{\Lambda\alpha}/\Lambda_H \simeq 0.4$.

We formally employ counting rules suggested by Bedaque and van Kolck for two-body systems in pionless EFT.^{34,35b} At LO the $\Lambda\Lambda$, Λd , or $\Lambda\alpha$ bubble diagram is resummed up to infinite order. The parameters in the renormalized composite propagators are determined by the scattering length and the binding momenta corresponding to the two-body binding energies. For the three-body systems, we employ the counting rules suggested by Bedaque, Hammer, and van Kolck.³⁷ When the system exhibits the limit cycle, the three-body contact interaction is promoted to LO in order to renormalize the cyclic singularity emerging in the system.

An effective Lagrangian consists of relevant degrees of freedom at low energies, for which the most general terms are constructed based on symmetries of the mother theory, and is perturbatively expanded in terms of the number of derivatives.³⁸ Because the typical momentum scales of the systems we consider are much smaller than the pion mass, the pions fields are considered to be heavy degrees of freedom.³⁹ In addition, instead of the chiral limit, the unitary limit can be chosen to describe the three-body systems,^{40,41} in which bound states of the three-body systems can appear as the Efimov states. While we do not take the unitary limit as LO in the present work, and thus all scattering lengths are finite. Explicit expressions of the effective Lagrangian for the $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ systems at LO are presented in Refs. 26 and 27, respectively.

2.2. Two-body part

Two two-body parts exist in each of the $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ three-body systems where we consider only S -waves for the two-body interactions at LO. Because the $\Lambda\Lambda$ part is common in both systems, three two-body parts, $\Lambda\Lambda$ in 1S_0 channel, Λd in ${}^3_\Lambda\text{H}$ channel, and $\Lambda\alpha$ in ${}^5_\Lambda\text{He}$ channel, are relevant. The renormalized dressed composite propagators for the two-body parts are presented below.

The diagrams for the dressed $\Lambda\Lambda$ dibaryon propagator are depicted in Fig. 1. The dressed $\Lambda\Lambda$ dibaryon propagator at LO is obtained as^{26,37c}

^b Counting rules known as the KSW counting rules suggested by Kaplan, Savage, and Wise³⁶ are for two nucleons and pions in chiral EFT.

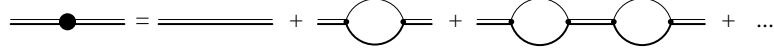


Fig. 2. Diagrams for dressed hypertriton propagator as the Λd system. In the right-hand side, the solid line denotes the Λ hyperon while the thick solid line represents the deuteron. The bare t field as a Λd composite state in hypertriton channel is denoted by the double (thin and thick) solid line.

$$D_s(p_0, \vec{p}) = \frac{4\pi}{m_\Lambda y_s^2} \frac{1}{\frac{1}{a_{\Lambda\Lambda}} - \sqrt{-m_\Lambda p_0 + \frac{1}{4}\vec{p}^2 - i\epsilon - i\epsilon}}, \quad (1)$$

where p_0 and \vec{p} are off-shell energy and three momentum, m_Λ is the Λ hyperon mass, $a_{\Lambda\Lambda}$ is the S -wave $\Lambda\Lambda$ scattering length, and y_s is the coupling constant of dibaryon to two Λ fields, $y_s = -\frac{2}{m_\Lambda} \sqrt{\frac{2\pi}{r_{\Lambda\Lambda}}}$; $r_{\Lambda\Lambda}$ is the effective range of the S -wave $\Lambda\Lambda$ scattering. We note that it is not necessary to have the effective range, $r_{\Lambda\Lambda}$, at LO. The $r_{\Lambda\Lambda}$ dependence in y_s disappears in on-shell quantities, such as a scattering amplitude, while it remains in intermediate quantities, such as $D_s(p_0, \vec{p})$.

In Fig. 2, diagrams for the dressed hypertriton propagator as the Λd system are depicted. The renormalized dressed hypertriton propagator is given as²⁶

$$D_t(p_0, \vec{p}) = \frac{2\pi}{\mu_{\Lambda d} y_t^2} \frac{1}{\gamma_{\Lambda d} - \sqrt{-2\mu_{\Lambda d} \left(p_0 - \frac{1}{2(m_d + m_\Lambda)} \vec{p}^2 \right) - i\epsilon - i\epsilon}}, \quad (2)$$

where $\mu_{\Lambda d}$ is the reduced mass of the Λd system, m_d is the deuteron mass, $\gamma_{\Lambda d}$ is the binding momentum of the hypertriton. In addition, y_t is the coupling constant of bare composite hypertriton field to the Λ hyperon and the deuteron fields, $y_t = -\frac{1}{\mu_{\Lambda d}} \sqrt{\frac{2\pi}{r_{\Lambda d}}}$; $r_{\Lambda d}$ is the effective range of the S -wave Λd scattering.^d

The expression of the dressed composite ${}^5_\Lambda\text{He}$ propagator of the $\Lambda\alpha$ system is similarly represented as that in Eq. (2) in terms of the reduced mass $\mu_{\Lambda\alpha}$, the α particle mass m_α , and the binding momentum $\gamma_{\Lambda\alpha}$ of ${}^5_\Lambda\text{He}$ as the $\Lambda\alpha$ system. One can find it in Eq. (8) in Ref. 27.

2.3. Three-body part

For the $\Lambda\Lambda d$ system, we derive integral equations for the S -wave Λ hyperon hypertriton scattering. Because the Λ hyperon and the hypertriton are both spin-1/2 there are two total spin states, spin-0 and 1, where we consider only S -waves between two constituents of the three-body system at LO.

In Fig. 3, diagrams for the S -wave Λ hyperon and hypertriton scattering in the spin-0 channel are depicted. We have a single integral equation in terms of the elastic scattering amplitude $t(p, k; E)$ in momentum space where $p = |\vec{p}|$ ($k = |\vec{k}|$)

^c In Refs. 42, 43 the effective range correction is resummed and included in the propagator.

^d See footnote f.

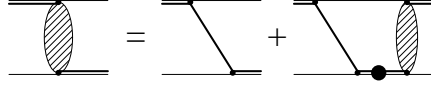
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Fig. 3. Diagrams of the integral equation for S -wave scattering of hypertriton and Λ for the spin singlet channel. See the caption of Fig. 2 as well.

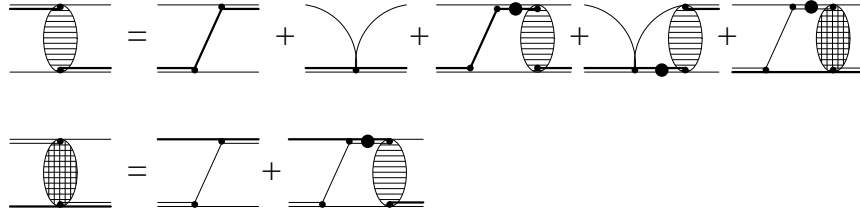


Fig. 4. Diagrams of coupled integral equations for S -wave scattering of hypertriton and Λ for spin triplet channel. See the captions of Figs. 1 and 2 as well.

is off-shell outgoing (on-shell incoming) relative three momentum in C.M. frame and E is the total energy of the system as²⁶

$$t(p, k; E) = -3K_{(a)}(p, k; E) + \frac{1}{2\pi^2} \int_0^{\Lambda_c} dl l^2 3K_{(a)}(p, l; E) D_t \left(E - \frac{\vec{l}^2}{2m_\Lambda}, \vec{l} \right) t(l, k; E), \quad (3)$$

where $K_{(a)}(p, l; E)$ is one-deuteron exchange interaction given as

$$K_{(a)}(p, l; E) = \frac{m_d y_t^2}{6pl} \ln \left(\frac{p^2 + l^2 + \frac{2\mu_{\Lambda d}}{m_d} - 2\mu_{\Lambda d} E}{p^2 + l^2 - \frac{2\mu_{\Lambda d}}{m_d} - 2\mu_{\Lambda d} E} \right), \quad (4)$$

and a sharp momentum cutoff Λ_c has been introduced in the equation. The total energy E for the scattering is $E = -\frac{\gamma_{\Lambda d}^2}{2\mu_{\Lambda d}} + \frac{1}{2\mu_{\Lambda(\Lambda d)}} k^2$ where $\mu_{\Lambda(\Lambda d)} = m_\Lambda(m_\Lambda + m_d)/(2m_\Lambda + m_d)$.

In Fig. 4, diagrams for the S -wave Λ hyperon and hypertriton scattering in spin-1 channel are depicted. We have coupled integral equations in terms of two amplitudes $a(p, k; E)$ and $b(p, k; E)$ where $a(p, k; E)$ is the amplitude in elastic channel and $b(p, k; E)$ is that in inelastic channel from the Λ hyperon and hypertriton cluster

channel to the $\Lambda\Lambda$ dibaryon and deuteron cluster channel as^{26 e}

$$\begin{aligned}
 a(p, k; E) &= K_{(a)}(p, k; E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \\
 &\quad - \frac{1}{2\pi^2} \int_0^{\Lambda_c} dl l^2 \left[K_{(a)}(p, k; E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \right] D_t \left(E - \frac{\vec{l}^2}{2m_\Lambda}, \vec{l} \right) a(l, k; E) \\
 &\quad - \frac{1}{2\pi^2} \int_0^{\Lambda_c} dl l^2 K_{(b1)}(p, k; E) D_s \left(E - \frac{\vec{l}^2}{2m_\Lambda}, \vec{l} \right) b(l, k; E), \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 b(p, k; E) &= K_{(b2)}(p, k; E) \\
 &\quad - \frac{1}{2\pi^2} \int_0^{\Lambda_c} dl l^2 K_{(b2)}(p, k; E) D_t \left(E - \frac{\vec{l}^2}{2m_\Lambda}, \vec{l} \right) a(l, k; E), \quad (6)
 \end{aligned}$$

where $g_1(\Lambda_c)$ is the coupling constant of the three-body contact interaction in the elastic channel. As will be seen below the system in this channel exhibits a limit cycle and the strength of the coupling $g_1(\Lambda_c)$ is fixed as a function of Λ_c . $K_{(b1)}(p, l; E)$ and $K_{(b2)}(p, l; E)$ are one- Λ hyperon exchange interactions given as

$$K_{(b1)}(p, l; E) = -\sqrt{\frac{2}{3}} \frac{m_\Lambda y_s y_t}{2pl} \ln \left(\frac{p^2 + \frac{m_\Lambda}{2\mu_{\Lambda d}} l^2 + pl - m_\Lambda E}{p^2 + \frac{m_\Lambda}{2\mu_{\Lambda d}} l^2 - pl - m_\Lambda E} \right), \quad (7)$$

$$K_{(b2)}(p, l; E) = -\sqrt{\frac{2}{3}} \frac{m_\Lambda y_s y_t}{2pl} \ln \left(\frac{\frac{m_\Lambda}{2\mu_{\Lambda d}} p^2 + l^2 + pl - m_\Lambda E}{\frac{m_\Lambda}{2\mu_{\Lambda d}} p^2 + l^2 - pl - m_\Lambda E} \right). \quad (8)$$

For the $\Lambda\Lambda\alpha$ system to describe the bound state of ${}^6_{\Lambda\Lambda}\text{He}$, we have similar coupled integral equations to those in Eqs. (5) and (6). Explicit expression of the equations can be found in Eq. (9) in Ref. 27.

3. Limit Cycle in the Systems

In this section, we derive equations in the asymptotic limit from the integral equations obtained in the previous section, and make a simple examination whether the systems exhibit a limit cycle or not. In the asymptotic limit, $l \sim p \gg E, k, 1/a_{\Lambda\Lambda}, \gamma_{\Lambda d}(\gamma_{\Lambda\alpha})$, the scales in the equations disappear and those equations have no scale dependence. The scale invariance in the asymptotic limit suggests that the amplitudes must exhibit a power-law behavior as³⁷

$$t(p), a(p) \sim p^{-1+s}, \quad (9)$$

where the k and E dependence in the amplitudes is dismissed. Then, by performing a Mellin transformation⁴⁴ to the homogeneous part of Eq. (3) and Eqs. (5,6) in the

^e Because the second integral equation in Eq. (6) depends only on $a(p, k; E)$, one can use it in the first integral equation in Eq. (5) leading to a single integral equation for $a(p, k; E)$.

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limit we obtain²⁷

$$1 = C_0 I_1(s), \quad (10)$$

$$1 = C_1 I_1(s) + C_2 I_2(s) I_3(s), \quad (11)$$

with

$$C_0 = -\frac{1}{2\pi} \frac{m_d}{\mu_{\Lambda d}} \sqrt{\frac{\mu_{\Lambda(\Lambda d)}}{\mu_{\Lambda d}}}, \quad (12)$$

$$C_1 = \frac{1}{6\pi} \frac{m_d}{\mu_{\Lambda d}} \sqrt{\frac{\mu_{\Lambda(\Lambda d)}}{\mu_{\Lambda d}}}, \quad C_2 = \frac{\sqrt{2}}{3\pi^2} \frac{\sqrt{m_{\Lambda} \mu_d (\Lambda \Lambda) \mu_{\Lambda(\Lambda d)}}}{\mu_{\Lambda d}^{3/2}}, \quad (13)$$

where $\mu_{d(\Lambda\Lambda)} = 2m_{\Lambda}m_d/(2m_{\Lambda} + m_d)$, and

$$I_1(s) = \frac{2\pi}{s} \frac{\sin[s \sin^{-1}(\frac{1}{2}a)]}{\cos(\frac{\pi}{2}s)}, \quad (14)$$

$$I_2(s) = \frac{2\pi}{s} \frac{1}{b^{s/2}} \frac{\sin[s \cot^{-1}(\sqrt{4b-1})]}{\cos(\frac{\pi}{2}s)}, \quad (15)$$

$$I_3(s) = \frac{2\pi}{s} b^{s/2} \frac{\sin[s \cot^{-1}(\sqrt{4b-1})]}{\cos(\frac{\pi}{2}s)}, \quad (16)$$

and $a = \frac{2\mu_{\Lambda d}}{m_d}$ and $b = \frac{m_{\Lambda}}{2\mu_{\Lambda d}}$. If the solutions of the equation in Eq. (10) or (11) are real, the physical solution of the amplitude should satisfy the condition that the half-off-shell amplitude converges in the asymptotic limit. On the other hand, if the solutions of the equation are complex, e.g., $s = \pm i s_0$, then the amplitude exhibits a cyclic behavior as $t(p), a(p) \sim e^{\pm i s_0 \ln(p)}$ in the limit where $p \rightarrow \infty$. Thus it is necessary to introduce a counter term to renormalize the cyclic divergence and fix the counter term by choosing a renormalization point.

The solutions of s in Eq. (10) are real and we have $s = \pm 2.0, \pm 2.0838 \dots, \pm 5.3227 \dots, \pm 6.8665 \dots, \dots$. Thus the $\Lambda\Lambda d$ system in the spin-0 channel will not exhibit a limit cycle and it is not necessary to introduce a three-body contact interaction for renormalization. It is expected that the system in this channel is not sensitive to a three-body contact interaction. The scattering amplitude for S -wave Λ hyperon and hypertriton scattering in the spin-0 channel may be well described by the two-body interaction at LO. The scattering length for the process is to be estimated in the next section.

The solutions of s in Eq. (11) are imaginary and we have $s = \pm i s_0$ where

$$s_0 = 0.4492 \dots. \quad (17)$$

In addition, the multiplicative factor of the discrete scaling symmetry becomes $e^{\pi/s_0} \simeq 1.09 \times 10^3$. Thus the $\Lambda\Lambda d$ system in the spin-1 channel will exhibit a limit cycle and the three-body contact interaction would be promoted to LO. However, it may not be easy to observe the cyclic pattern in the system because the typical scale is $\gamma_{\Lambda d} \simeq 13.5$ MeV and the next scale in the discrete scaling symmetry appears

at $\gamma_{\Lambda d}^{(2)} = e^{\pi/s_0} \gamma_{\Lambda d} \sim 15 \times 10^3$ MeV, which is much larger than the hard scale of the theory, $\Lambda_H \sim 50$ MeV.

In the case of the $\Lambda\Lambda\alpha$ system, on the other hand, we have the same equation as that in Eq. (11) but different values of the coefficients, C_1 and C_2 and the parameters a and b in the functions $I_1(s)$, $I_2(s)$, and $I_3(s)$ in Eqs. (14), (15), and (16),²⁷ which are

$$C_1 = \frac{1}{2\pi} \frac{m_\alpha}{\mu_{\Lambda\alpha}} \sqrt{\frac{\mu_{\Lambda(\Lambda\alpha)}}{\mu_{\Lambda\alpha}}}, \quad C_2 = \frac{\sqrt{2m_\Lambda \mu_{\Lambda(\Lambda\alpha)} \mu_{\alpha(\Lambda\Lambda)}}}{\pi^2 \mu_{\Lambda\alpha}^{3/2}}, \quad (18)$$

where $\mu_{\Lambda(\Lambda\alpha)} = m_\Lambda(m_\Lambda + m_\alpha)/(2m_\Lambda + m_\alpha)$ and $\mu_{\alpha(\Lambda\Lambda)} = 2m_\Lambda m_\alpha/(2m_\Lambda + m_\alpha)$, and $a = 2\mu_{\Lambda\alpha}/m_\alpha$ and $b = m_\Lambda/(2\mu_{\Lambda\alpha})$. The solutions of the equation are imaginary as well and we have $s = \pm i s_0$ where

$$s_0 = 1.0496 \dots \quad (19)$$

The multiplicative factor becomes $e^{\pi/s_0} \simeq 19.9$, which is similar to that in the case of three nucleons in triton channel, $e^{\pi/s_0} \simeq 22.7$, where $s_0 \simeq 1.00624$.^{45,46} Thus the $\Lambda\Lambda\alpha$ system in the spin-0 channel will exhibit a limit cycle and a three-body contact interaction should be included at LO. The typical scale of the system is $\gamma_{\Lambda\alpha} \simeq 73.2$ MeV and the next scale in the discrete scaling symmetry will appear at $\gamma_{\Lambda\alpha}^{(2)} = e^{\pi/s_0} \gamma_{\Lambda\alpha} \simeq 1.5 \times 10^3$ MeV, which is about 10 times larger than the hard scale of the theory, $\Lambda_H \simeq 170$ MeV. Thus it would not be easy to see the second one of the discrete scaling symmetry in the $\Lambda\Lambda\alpha$ system either. Though it may be difficult to observe the cyclic pattern in the systems, the first bound state appearing in the limit cycle may correspond to those of ${}_{\Lambda\Lambda}^4\text{H}$ and ${}_{\Lambda\Lambda}^6\text{He}$. That is to be studied in the next section.

4. Numerical Results

In this section we review our numerical results presented in Refs. 26, 27. We discuss the numerical results for the scattering length of the S -wave Λ hyperon and hypertriton in the spin-0 channel, in which a limit cycle does not appear, and for the bound states of ${}_{\Lambda\Lambda}^4\text{H}$ and ${}_{\Lambda\Lambda}^6\text{He}$ as the $\Lambda\Lambda d$ system in the spin-1 channel and the $\Lambda\Lambda\alpha$ system in the spin-0 channel, in which the limit cycle appears.

4.1. Scattering channel not exhibiting limit cycle

The integral equation at LO in Eq. (3) can be fixed by the masses and the two effective range parameters, $\gamma_{\Lambda d}$ and $r_{\Lambda d}$, where $\gamma_{\Lambda d} = 13.5$ MeV and $r_{\Lambda d} = 2.3 \pm 0.3$ fm.^{47 f} The scattering length a_0 of the S -wave Λ hyperon and hypertriton

^f As mentioned above, it is not necessary to have the effective range, $r_{\Lambda d}$, at LO. In our calculation, the $r_{\Lambda d}$ dependence indeed disappears in the scattering amplitude, $T(k, k)$, while it remains in the intermediate quantities, such as $K_{(a)}(p, k; E)$, $D_t(p_0, \vec{p})$, and $Z_{\Lambda d}$.

scattering in the spin-0 channel is calculated by using the formulae,

$$a_0 = -\frac{\mu_{\Lambda(\Lambda d)}}{2\pi} T(0, 0). \quad (20)$$

In the expression above, $T(k, k)$ is the on-shell scattering amplitude given by

$$T(k, k) = \sqrt{Z_{\Lambda d}} t(k, k; E) \sqrt{Z_{\Lambda d}}, \quad (21)$$

where $Z_{\Lambda d}$ is the wavefunction normalization factor of the hypertriton as the Λd system, $Z_{\Lambda d} = \gamma_{\Lambda d} r_{\Lambda d}$. Thus we obtain²⁶

$$a_0 = 16.0 \pm 3.0 \text{ fm}. \quad (22)$$

No experimental datum for the quantity is available because there are no enough hypertriton targets for experiment. The phase shift up to the hypertriton breakup threshold is also calculated and displayed in Fig. 6 in Ref. 26.

4.2. *Binding channels exhibiting limit cycle*

4.2.1. ${}^4_{\Lambda\Lambda}H$ as $\Lambda\Lambda d$ system

The $\Lambda\Lambda d$ system in the spin-1 channel described by the coupled integral equations at LO can be determined by three constants, $\gamma_{\Lambda d}$, $a_{\Lambda\Lambda}$, and $g_1(\Lambda_c)$. As mentioned, $\gamma_{\Lambda d} = 13.5 \text{ MeV}$ whereas the value of $a_{\Lambda\Lambda}$ is experimentally deduced from the ${}^{12}\text{C}(K^-, K^+ \Lambda\Lambda X)$ reaction,⁴⁸ which leads to $a_{\Lambda\Lambda} = -1.2 \pm 0.6 \text{ fm}$.⁴⁹ In addition, the data for the Au+Au collisions at the Relativistic Heavy Ion Collider⁵⁰ are analyzed and $a_{\Lambda\Lambda} \geq -1.25 \text{ fm}$ is reported in Ref. 51. To fix the coefficient of the three-body contact interaction $g_1(\Lambda_c)$ one needs to have some three-body datum, but there are no available experimental data at the present moment. We make use of the results from the model calculations in which the formation of the ${}^4_{\Lambda\Lambda}H$ bound state is reported.^{21,22} We employ three set of the parameters of $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$:

$$\text{(I)} \quad B_{\Lambda\Lambda} \simeq 0.2 \text{ MeV and } a_{\Lambda\Lambda} \simeq -0.5 \text{ fm}, \quad (23)$$

$$\text{(II)} \quad B_{\Lambda\Lambda} \simeq 0.6 \text{ MeV and } a_{\Lambda\Lambda} \simeq -1.5 \text{ fm}, \quad (24)$$

$$\text{(III)} \quad B_{\Lambda\Lambda} \simeq 1.0 \text{ MeV and } a_{\Lambda\Lambda} \simeq -2.5 \text{ fm}. \quad (25)$$

In Fig. 5, we plot the curves of $g_1(\Lambda_c)$ as functions of the cutoff Λ_c where $g_1(\Lambda_c)$ is fixed by using the three sets of the parameters labeled by (I), (II), and (III) introduced above. One can see that the curves of $g_1(\Lambda_c)$ are rather mildly varying at $\Lambda_c = 10 \sim 10^4 \text{ MeV}$, and each curve has a singularity at $\Lambda_c \sim 10^5 \text{ MeV}$ indicating the possibility of the first cycle of the limit-cycle. This implies that the one-deuteron-exchange and one- Λ -exchange interactions for the spin-1 channel contains an attractive (singular) interaction at very high momentum, say, $\Lambda_c \sim 10^5 \text{ MeV}$. At such a very high momentum, however, the applicability of the present theory, a very low energy EFT, cannot be guaranteed and thus the mechanisms of the formation of a bound state must have different origins. We note, on the other hand, that, if we choose $g_1(\Lambda_c) \simeq -2$ or smaller at $\Lambda_c \sim 50 \text{ MeV}$ in the coupled

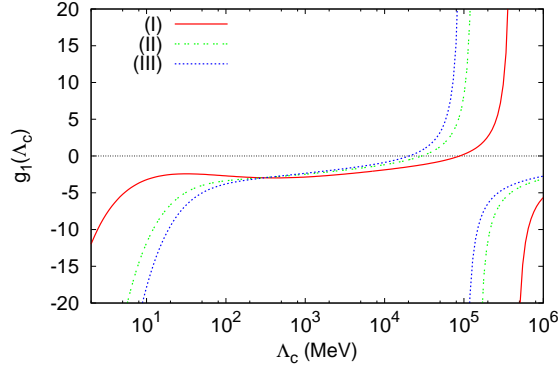


Fig. 5. Coupling $g_1(\Lambda_c)$ of three-body contact interaction as functions of the cutoff Λ_c which produces a bound state of ${}^4_{\Lambda\Lambda}\text{H}$ with three different sets of $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$. See the text for the parameter sets (I), (II), and (III).

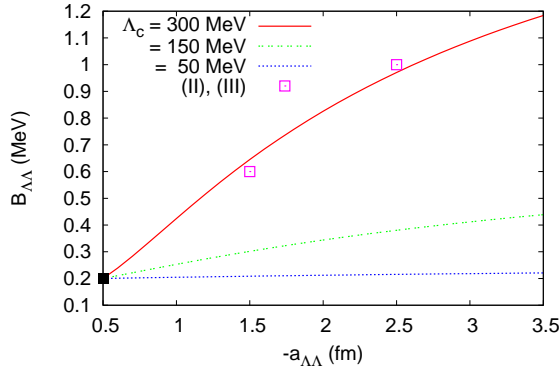


Fig. 6. Calculated two- Λ separation energy $B_{\Lambda\Lambda}$ from ${}^4_{\Lambda\Lambda}\text{H}$ bound state as functions of the scattering length $a_{\Lambda\Lambda}$ of the S -wave $\Lambda\Lambda$ scattering for the 1S_0 channel with the cutoff values $\Lambda_c = 50, 150, 300$ MeV. The value of $g_1(\Lambda_c)$ of all three curves is fitted at the point (I): $B_{\Lambda\Lambda} = 0.2$ MeV and $a_{\Lambda\Lambda} = -0.5$ fm, marked by a filled square. The points (II) and (III) are also included as blank squares in the figure.

integral equations, a bound state can be created. Such a value of $g_1(\Lambda_c)$ is of natural size and may be generated from the mechanisms of high energy such as σ -meson exchange or two-pion exchange near the intermediate range of nuclear force, i.e., $\Lambda_c = 300 \sim 600$ MeV.

In Fig. 6, we plot the curves of $B_{\Lambda\Lambda}$ as functions of $a_{\Lambda\Lambda}$. Here, the coupling $g_1(\Lambda_c)$ is fixed by using the parameter set (I), i.e., $B_{\Lambda\Lambda} = 0.2$ MeV and $a_{\Lambda\Lambda} = -0.5$ fm, which is marked by a filled square in Fig. 6. This is achieved with $g_1(\Lambda_c) \simeq -2.48$,

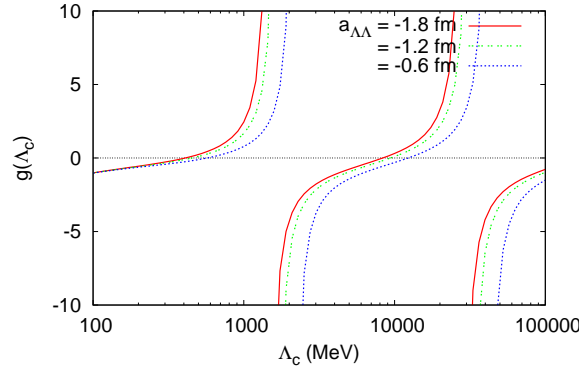


Fig. 7. The coupling $g(\Lambda_c)$ as functions of Λ_c for $a_{\Lambda\Lambda} = -0.6, -1.2, -1.8$ fm where the values of $g(\Lambda_c)$ are fitted by $B_{\Lambda\Lambda} = 6.93$ MeV of ${}_{\Lambda\Lambda}^6\text{He}$.

$-2.83, -2.96$ for $\Lambda_c = 50, 150, 300$ MeV, respectively. Once the starting values are fixed, we vary the value of $a_{\Lambda\Lambda}$ for a fixed value of Λ_c , which changes the values of $B_{\Lambda\Lambda}$. We then find that the behaviors of the $B_{\Lambda\Lambda}$ curves as functions of $a_{\Lambda\Lambda}$ are quite sensitive to the values of the cutoff Λ_c . For example, when we choose $\Lambda_c \simeq \Lambda_H$, i.e., $\Lambda_c = 50$ MeV, $B_{\Lambda\Lambda}$ is insensitive to the value of $a_{\Lambda\Lambda}$ and makes a nearly flat curve as shown by the dotted line in Fig. 6. However, with a larger cutoff value, $\Lambda_c = 300$ MeV, $B_{\Lambda\Lambda}$ strongly depends on $a_{\Lambda\Lambda}$ and we can fairly well reproduce the $a_{\Lambda\Lambda}$ -dependence of $B_{\Lambda\Lambda}$ obtained in the potential model calculation by Filikhin and Gal²¹ or Nemura *et al.*²²

4.2.2. ${}_{\Lambda\Lambda}^6\text{He}$ as $\Lambda\Lambda\alpha$ system

The $\Lambda\Lambda\alpha$ system in the spin-0 channel described by the coupled integral equations at LO can also be determined by three coupling constants, $\gamma_{\Lambda\alpha}$, $a_{\Lambda\Lambda}$, and $g(\Lambda_c)$, where $g(\Lambda_c)$ is a coupling constant of a three-body contact interaction for the $\Lambda\Lambda\alpha$ system.²⁷ As mentioned above, the Λ separation momentum of ${}_{\Lambda}^5\text{He}$ is given by $\gamma_{\Lambda\alpha} \simeq 73.2$ MeV, there is the uncertainty in $a_{\Lambda\Lambda}$, and $g(\Lambda_c)$ can be fixed by using the experimental data of $B_{\Lambda\Lambda}$ from ${}_{\Lambda\Lambda}^6\text{He}$, $B_{\Lambda\Lambda} = 6.93 \pm 0.16$ MeV.

In Fig. 7, we plot the curves of $g(\Lambda_c)$ as functions of the cutoff Λ_c with three different values of $a_{\Lambda\Lambda} = -1.8, -1.2, -0.6$ fm so as to reproduce the experimental data of $B_{\Lambda\Lambda}$. The curves are numerically obtained from the homogeneous part of the coupled integral equations in Eq. (9) in Ref. 27. One can see that the curves clearly exhibit a limit cycle and the first divergence appears at $\Lambda_c \sim 1$ GeV. In addition, a larger value of $|a_{\Lambda\Lambda}|$ behaves as giving a larger attractive force and shifts the curves of $g(\Lambda_c)$ to the left in Fig. 7.

The value of s_0 obtained in Eq. (19) can be estimated from the curves of the limit cycle of $g(\Lambda_c)$ in Fig. 7. The $(n+1)$ -th values of Λ_n at which $g(\Lambda_c)$ vanishes

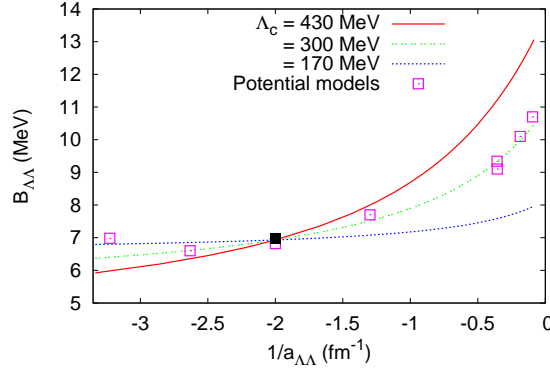


Fig. 8. The two- Λ separation energy $B_{\Lambda\Lambda}$ as functions of $1/a_{\Lambda\Lambda}$ for $\Lambda_c = 170, 300, 430$ MeV, where $g(\Lambda_c)$ is renormalized at the point of $B_{\Lambda\Lambda} = 6.93$ MeV and $1/a_{\Lambda\Lambda} = -2.0$ fm $^{-1}$ that is marked by a filled square. Open squares are the results from the potential models in Table 5 of Ref.²⁰

can be parameterized as $\Lambda_n = \Lambda_0 \exp(n\pi/s_0)$ in the discrete scaling symmetry. By using the second and third values of Λ_n for the three values of $a_{\Lambda\Lambda}$, we have $s_0 = \pi/\ln(\Lambda_2/\Lambda_1) \simeq 1.05$, which is in a very good agreement with the value of Eq. (19). Furthermore, the value of s_0 may be checked by using Fig. 52 in Ref. 32 which is a plot of $\exp(\pi/s_0)$ versus m_1/m_3 for the mass-imbalanced system where $m_1 = m_2 \neq m_3$. In our case, $m_1/m_3 = m_\Lambda/m_\alpha \simeq 0.3$, one finds $\exp(\pi/s_0) \simeq 20$ from the figure and thus $s_0 \simeq 1.05$. This is another very good agreement with what we find in Eq. (19).

In Fig. 8, we plot the curves of $B_{\Lambda\Lambda}$ as functions of $a_{\Lambda\Lambda}$ where $g(\Lambda_c)$ is renormalized at the point (marked by the filled square in the figure) of $B_{\Lambda\Lambda} = 6.93$ MeV and $1/a_{\Lambda\Lambda} = -2.0$ fm $^{-1}$ with three reference values of the cutoff $\Lambda_c = 430, 300, 170$ MeV. This leads to $g(\Lambda_c) \simeq -0.715, -0.447, -0.254$ for $\Lambda_c = 170, 300, 430$ MeV, respectively. Open squares are the estimated values from the potential models given in Table 5 of Ref. 20. We find that the curves are sensitive to the cutoff values. When we choose $\Lambda_c = \Lambda_H \simeq 170$ MeV, the curve for $B_{\Lambda\Lambda}$ is found to be rather insensitive to $1/a_{\Lambda\Lambda}$. While the results from the potential models are remarkably well reproduced by the curve with $\Lambda_c = 300$ MeV.

5. Discussion and conclusions

In this article we reviewed the results of our recent works on the light double Λ hypennuclei, ${}_{\Lambda\Lambda}^4\text{H}$ and ${}_{\Lambda\Lambda}^6\text{He}$, as the $\Lambda\Lambda d$ and $\Lambda\Lambda\alpha$ three-body systems investigated in halo/cluster EFT at LO.^{26,27} We obtained the equations from the integral equations in the asymptotic limit to make a simple examination whether the systems exhibit a limit cycle or not. When a limit cycle does not appear in the three-body system,

it is not necessary to introduce a three-body contact interaction and the system can be described by the two-body interaction. Thus we estimate the scattering length a_0 for the S -wave Λ hyperon and hypertriton scattering in the spin-0 channel without an unknown parameter. Our theoretical prediction is $a_0 = 16.0 \pm 3.0$ fm. Though our result cannot be tested due to the lack of any experimental datum, the similar case, the S -wave nd scattering in the spin-quartet channel, may give a positive indication that the theory has a prediction power in the channel without exhibiting the limit cycle. In the three-nucleon system a limit cycle does not appear either and the scattering length and the phase shift below the deuteron breakup threshold are well described without a three-body contact interaction.^{34, 52} The scattering length a_4 is obtained in pionless EFT as $a_4 = 6.33$ fm³⁴ which is in good agreement with the experimental value of $a_4^{exp.} = 6.35 \pm 0.02$ fm.⁵³

When the limit cycle appears in the system, it is necessary to introduce the three-body contact interaction for renormalization, and the bound states appearing in the system may sometimes be regarded as an Efimov state. For the $\Lambda\Lambda d$ system in the spin-1 channel, as seen in Fig. 5, the bound state of ${}_{\Lambda\Lambda}^4\text{H}$ can be formed due to the three-body contact interaction, which stems from the physics at the high energies, in the range of the cutoff value up to $\Lambda_c \sim 10^4$ MeV. The bound state can be formed without the three-body contact interaction when the cutoff value becomes larger than $\Lambda_c \sim 2 \times 10^4$ MeV. The value is extremely large compared to the hard scale of the $\Lambda\Lambda d$ system, $\Lambda_H \simeq 50$ MeV. Thus it is hard to regard that of ${}_{\Lambda\Lambda}^4\text{H}$ as an Efimov state.

On the other hand, the bound state of the $\Lambda\Lambda\alpha$ system, ${}_{\Lambda\Lambda}^6\text{He}$, can be formed by choosing the value of the cutoff as $\Lambda_c = 450 \sim 600$ MeV without introducing the three-body contact interaction where the range of the cutoff dependence comes out of the uncertainty of $a_{\Lambda\Lambda}$. That is comparable to the hard scale of the system, $\Lambda_H \simeq 170$ MeV. Thus the bound state for ${}_{\Lambda\Lambda}^6\text{He}$ formed at the first cycle of the limit cycle would be regarded as an Efimov state. Another example for the formation of an Efimov state in the three-body system is the triton. In pionless EFT calculations, the bound state of the three-nucleon system is formed without introducing the three-body contact interaction when one chooses $\Lambda_c \simeq 380$ MeV.⁴⁶ That is also comparable to the hard scale of the pionless EFT, $\Lambda_H = m_\pi \simeq 140$ MeV.

As discussed in the introduction, the systems can be described by a small number of the parameters in the very low energy EFTs. There is only one parameter for the $\Lambda\Lambda d$ system in the spin-0 channel, whereas there are three for the $\Lambda\Lambda d$ system in the spin-1 channel and the $\Lambda\Lambda\alpha$ system in the spin-0 system. In addition, each of the three-body systems is characterized by the typical and hard scales and the presence or absence of the limit cycle. This is a simple analysis but may provide us nontrivial information and knowledge about the systems. It may be interesting to apply the analysis carried out in the present article to the study of an exotic bound state arising from other three-body systems^g.

^g Recently, we applied the analysis to the study of a bound state formation in the $nn\Lambda$ system.⁵⁴

It may be worth discussing the role of the cutoff value in the systems.^h As mentioned, the correlations between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$ obtained from the potential models are well reproduced in both the ${}^4_{\Lambda\Lambda}\text{H}$ and ${}^6_{\Lambda\Lambda}\text{He}$ states when we choose $\Lambda_c = 300$ MeV. It may indicate that our simple theory can probe the scale of the $\Lambda\Lambda$ interactions of the potential model calculations. The value of Λ_c is consistent to the scale for the long range part of the $\Lambda\Lambda$ interaction which consists of the two-pion-exchange, $2m_\pi \sim 300$ MeV. With such a momentum cutoff, however, the two nucleon structure of the deuteron and the existence of the excited state of the α particle can be probed in the loops. Because our EFTs at very low energies do not have such detailed structures, the short range structural mechanism is missing in that case. Such an inconsistency is in fact common in the cluster model calculations.

When we strictly choose the cutoff Λ_c to be the hard scale of the systems, $\Lambda_c = \Lambda_H \simeq 50$ MeV for the $\Lambda\Lambda d$ system and $\Lambda_c = \Lambda_H \simeq 170$ MeV for the $\Lambda\Lambda\alpha$ system, on the other hand, the curves for $B_{\Lambda\Lambda}$ are less sensitive to $a_{\Lambda\Lambda}$, compared to the potential models, in Figs. 6 and 8, respectively. The contribution from the $\Lambda\Lambda$ two-body part with the different cutoff value is eventually compensated by the three-body contact interaction after the renormalization of the three-body quantity in each of the systems. In addition, the scale of $1/a_{\Lambda\Lambda}$ is $|1/a_{\Lambda\Lambda}| = 330 \sim 110$ MeV, which is comparable to the hard scales of the systems. Thus some portion of the $\Lambda\Lambda$ interaction is carried by the three-body contact term in the very low energy EFTs and is not easily disentangled from it.

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^h A value of the cutoff Λ_c can be arbitrary when renormalizing an on-shell physical quantity of the three-body system. (While we may take the value of Λ_c to be smaller than a value at which an unphysical deeply bound state is formed.) In this work, to assure the consistency between the effective degrees of freedom and the off-shell probe of the loop momentum, we assume $\Lambda_c \sim \Lambda_H$.

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